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## **Evolution of solitary one-humped three-dimensional waves on vertically falling liquid films**

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### **Abstract**

The results of the experimental study of 3-D waves evolving from a localized disturbance on a vertically falling liquid film in the range of film flow Reynolds numbers  $1 < Re < 25$  for three liquids with different physical properties are presented. Wave patterns were registered using LIF method which allows measuring both instantaneous shape and velocity of evolving 3-D waves. Two scenarios of 3-D solitary wave evolution were observed: formation of wave train and evolution in form of one-humped solitary wave. For several investigated flow regimes stationary solitary one-humped 3-D waves were registered. Characteristics of registered stationary 3-D waves for low  $Re$  are close to the theoretically predicted ones.

### **Introduction**

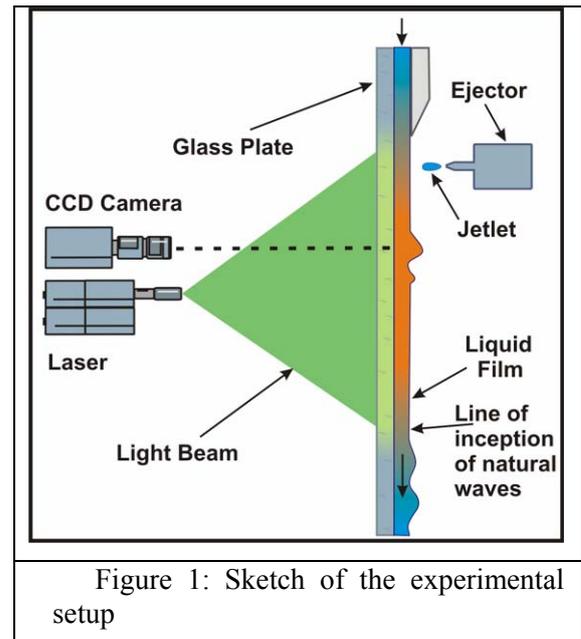
The regime of three-dimensional solitary waves is the last stage of wave evolution on a falling liquid film at low and moderate Reynolds numbers. On this stage all the film surface is covered by numerous nonlinear horseshoe-shaped waves, which propagate on thin residual layer and interact with each other in a random fashion [1]. This interaction hampers experimental investigation of 3-D waves. One of the possible ways to study in experiments deterministic characteristics of solitary 3-D waves is to generate them by a localized source in the waveless region of film flow. Such region is observed at any  $Re$  numbers in the upper part of film flow, where magnitudes of the natural waves are small and film can be considered as smooth one. Such approach was previously used by authors to study the evolution of localized disturbances in film flow at low Reynolds numbers [2]. In addition to previous experimental results on solitary 3-D waves evolution at low Reynolds numbers in this letter we present additional results for film flows with Reynolds numbers  $2 < Re < 25$ . Here  $Re=q/v$ , where  $q$  is the specific volumetric flow rate of liquid,  $v$  is the kinematic viscosity. As in the case of

smaller  $Re$ , two different ways of evolution exist for this range of  $Re$ . For  $Re < 5$  characteristics of the registered stationary 3-D waves well agree with theory.

### Experimental Facility

Experiments were carried out on a vertical glass plate with dimensions of 20 cm in transversal and 30 cm in vertical directions. Film flow was formed by a slot distributor with the slot width of 0.2 mm.

LIF method was used to measure instantaneous local film thickness on the area of  $10 \times 10$  cm with spatial resolution 0.1 mm. In general, the method is similar to fluorescence imaging [3, 4] and is based on the reconstruction of local film thickness in accordance with local brightness of fluorescence emitted by a small amount of fluorescent dye dissolved in working liquid. In our experiments standard PIV equipment with some modifications of beam former to eliminate speckles was used as measuring unit.



Pulsed doubled Nd:YAG laser with wave length of 532 nm was used as a light source for fluorescence excitation and a double-frame mode CCD camera with low-pass filter ( $> 550$  nm) was used for registration. This setup allows measuring not only instantaneous shape but also instantaneous velocity of quickly evolving 3-D waves. Both the light source and the camera were mounted from the dry side of the glass plate, i.e. both excitation and registration was carried out through glass layer.

Such arrangement allows minimizing optical distortions on the wavy boundary between air and working liquid. The following working fluids were used in experiments:

- alcohol-water solution with the density  $\rho = 931 \text{ kg/m}^3$ , surface tension  $\sigma = 0.03 \text{ kg/s}^2$ , kinematic viscosity  $\nu = 2.7 \cdot 10^{-6} \text{ m}^2/\text{s}$ ;
- water-glycerol solution with  $\rho = 1070 \text{ kg/m}^3$ ,  $\sigma = 0.065 \text{ kg/s}^2$ ,  $\nu = 2.2 \cdot 10^{-6} \text{ m}^2/\text{s}$ ;
- water-glycerol solution with  $\rho = 1140 \text{ kg/m}^3$ ,  $\sigma = 0.070 \text{ kg/s}^2$ ,  $\nu = 6.5 \cdot 10^{-6} \text{ m}^2/\text{s}$ .

Rhodamine 6G, which is not surfactant, was used as fluorescent dye with concentration of 0.01% wt. 3-D waves were excited by a short (with duration of 10 – 15 ms), impact of thin jetlet of working liquid in the upper part of the waveless region of film flow. The jetlet was

ejected from the nozzles with diameters of 0.15-1.33 mm. During the experiments, values characterizing the energy of excitation (velocity and mass of the jetlet) varied significantly.

When pulsed laser is used for excitation of fluorescence, liquid film is illuminated by weakly divergent beam of monochromatic light. This feature allows one to simplify the process of calibration and perform simple computation of errors connected with optical distortions on curvilinear free surface of the flowing film.

Calibration was carried out on the flat layers of working liquid. The layers were formed in the gaps with fixed width between the surface of the working plate and auxiliary glass plate. Preliminary investigation has shown that in this case local intensity  $J(x, y)$  and local film thickness  $h(x, y)$  are connected in the following way:

$$J(x, y) = C(x, y) \times [1 - \exp(-\alpha \cdot h(x, y))] \times [1 + K(x, y) \times \exp(-\alpha \cdot h(x, y))] + D(x, y)$$

Here  $\alpha$  – is the absorption coefficient for incident (exciting) light,  $K(x, y)$  – is the reflection index of remote surface,  $D(x, y)$  – is the dark level of the CCD camera. Absorption coefficient is a physical characteristic of the working solution and can be determined independently of the experiment conditions. The transfer function  $C(x, y)$  which is required for local film thickness reconstruction can be determined by single flat layer with fixed depth when absorption coefficient of the working solution is known. Calibration in our experiments was performed using two flat layers of working liquid with sufficiently different depths. One of the layers was used for determination of  $C(x, y)$  and the other for validation of the calibration.

Main errors of the experiments are connected with camera noise and little spatial redistribution of light intensity for consecutive laser pulses which results in total error from 3 to 7 microns for films with thickness between 200 and 500 microns. Additional sources of errors are caused by thermal drift of the camera which results in error less than 1% for long-run experiments and with optical distortions due to redistribution of reflected light on the wavy boundary between working liquid and air. Use of weakly divergent beam of exciting light makes it possible to carry out, in geometric optics approximation, simple computation of errors related to redistribution of exciting and emitted light intensity under the curvilinear free surface of the film. At computation the reconstructed form of the film surface is taken as true, and the dependence of reflection index for the light on the angle of inclination of the free liquid surface as well as the redistribution of intensity inside the liquid layer due to focusing (defocusing) under curvilinear segments of free boundary, are accounted for. Study of different experimental scenarios showed that main optical distortions are always connected

with focusing under the humps of large surface waves, and for all experimentally registered waves this distortions result in error significantly smaller than 1%.

## Results and Discussion

As in the case of previous investigation for small  $Re$  [2], for moderate Reynolds number relatively low level of excitation energy leads to quick growth of the amplitude of initial pulse

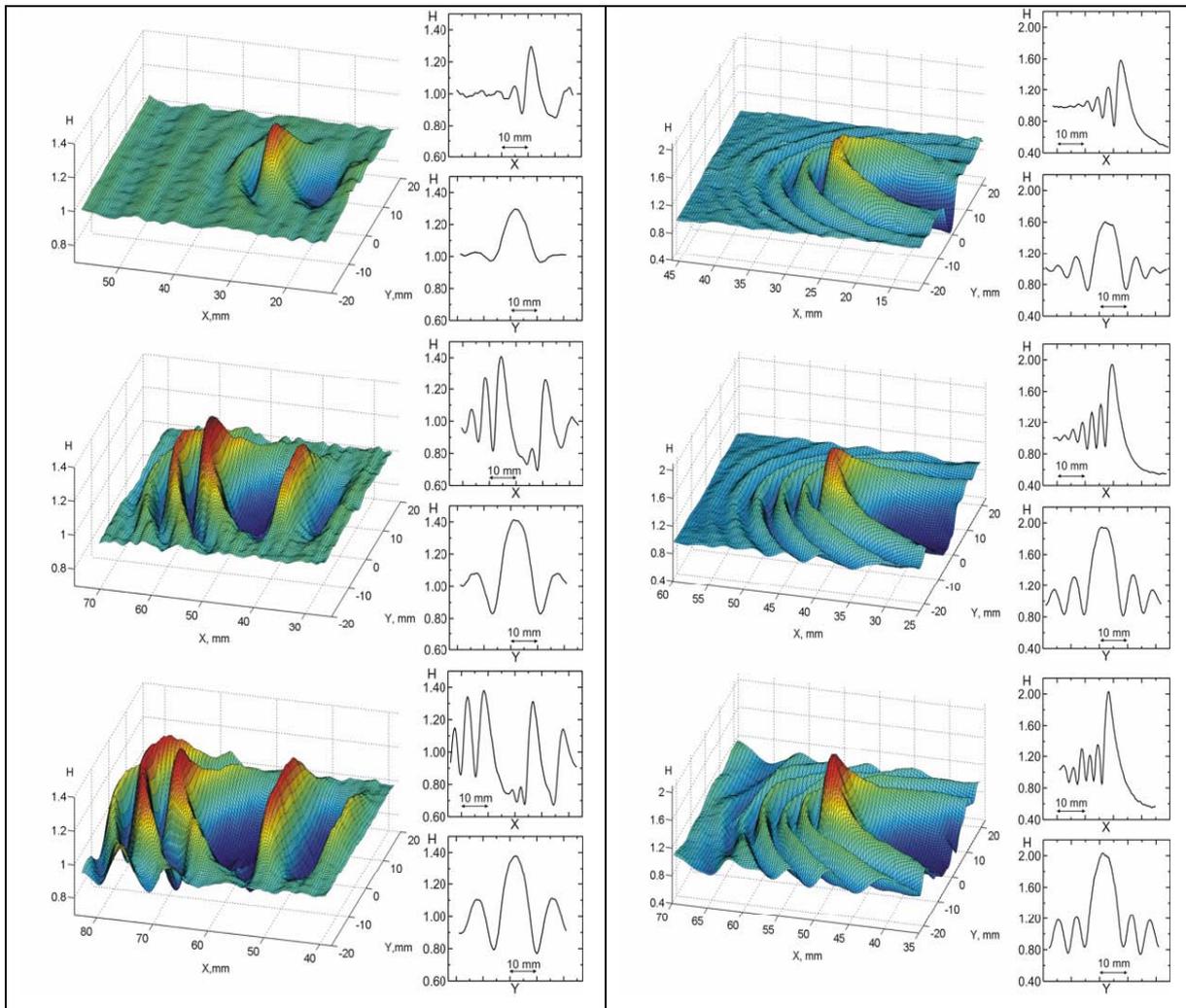


Figure 2: Development of the wave train at consecutive moments. On the right, longitudinal and transverse sectional views of the waveform are shown in the planes going through the wave maximum. Working liquid is water-glycerol solution ( $\nu = 2.2 \cdot 10^{-6} \text{ m}^2/\text{s}$ ),  $Re = 12$ .  $X$  is downstream distance from point of excitation.

Figure 3: Evolution in the form of a one-humped non-stationary solitary wave. Working liquid is water-alcohol solution,  $Re = 25$ .

and to subsequent formation of a wave train. In contrast to previous results, when linear growth of main hump of train continued right up to the region of developed natural waves, for

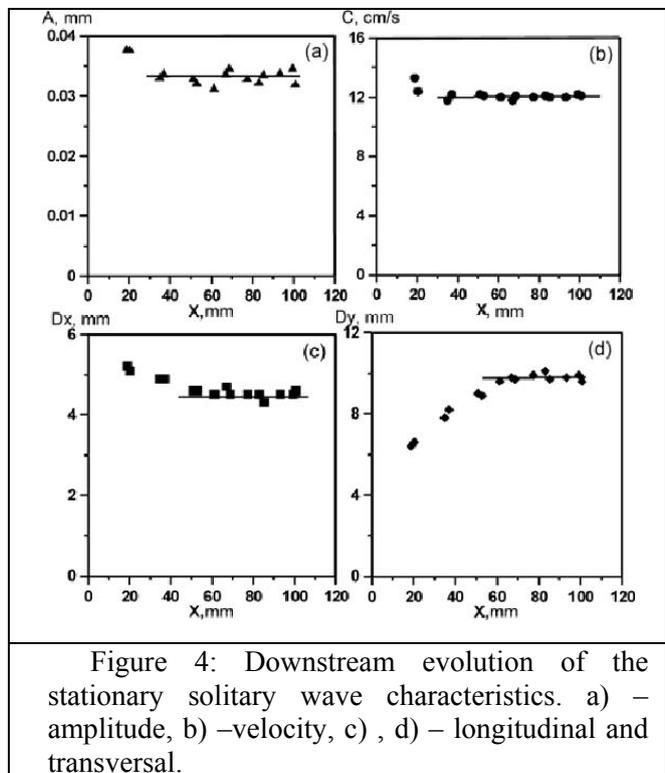
larger  $Re$  main hump saturates within waveless region of film flow without termination of train generation, as shown in Fig. 2. Here and further  $H$  is the wave amplitude divided by Nusselt flat film thickness.

This confirms our assumption that one-humped solitary 3-D wave can be excited only when initial distribution has the energy higher than some threshold value.

When the level of excitation is sufficiently high, an initial pulse is developing in the form of a one-humped solitary wave which always has horseshoe form and is usually non-stationary, as shown in Fig 3.

Special effort was made to find stationary 3-D waves. It was assumed that there exist some conditions of excitation at which initial disturbances can evolve into stationary three-dimensional solitary wave very fast. Therefore, the energy of excitation was varied in a wide range of values. Then the evolution of wave characteristics, such as amplitude, velocity, half-width of main hump in longitudinal direction and half-width of horseshoe in transversal dimensions was analyzed. The wave was considered to be stationary if only all of the above-mentioned parameters were unchanged in the lower part of waveless region at the distance equal to at least three specific longitudinal dimensions as shown in Fig. 4 for stationary solitary wave with longitudinal half-width equal to 4.5 mm.

It should be noted that there is no clear understanding about domain of attraction to the stationary wave from initial perturbation with characteristics sufficiently different from those of the wave. Amplitude and velocity of the initial perturbation were changing most often in the process of evolution in the experiments, though sometimes very slowly. But in some of the experimental runs waves were registered for which three of four characteristics, for example amplitude, velocity and longitudinal half-width reach stationary state, whereas the fourth characteristic, in this example it is the half-width of horseshoe, remains developing rapidly. For the above-mentioned example both trends, widening as well as contraction of the horseshoe were registered. So only in



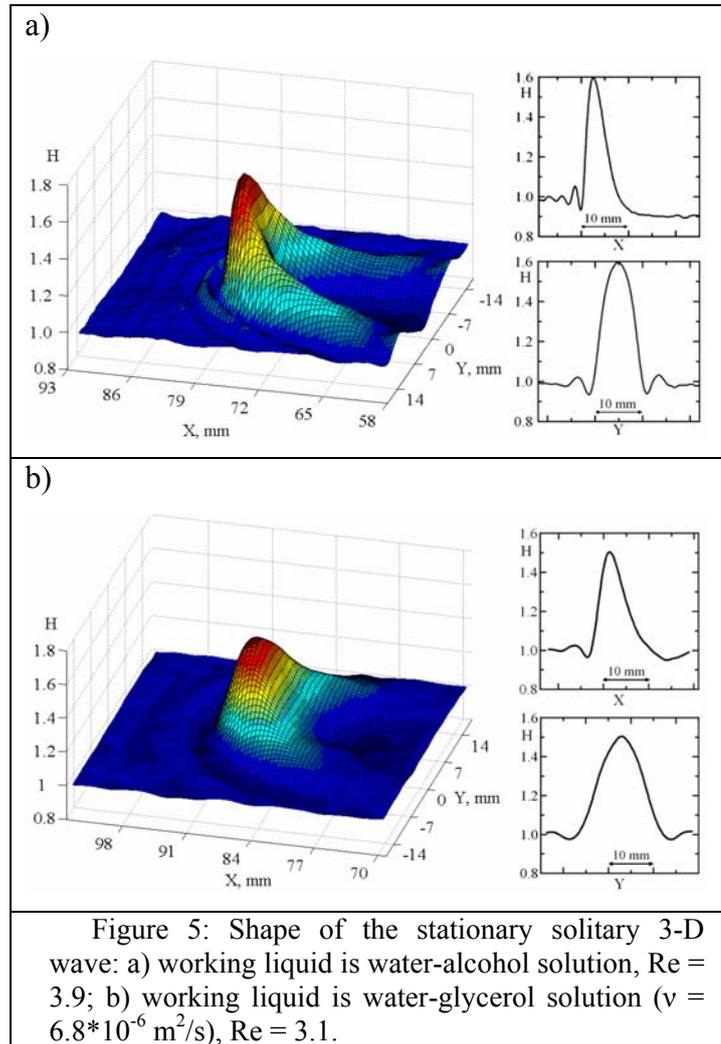
several experimental conditions stationary 3-D solitary waves were registered and mostly at low Reynolds numbers of film flow.

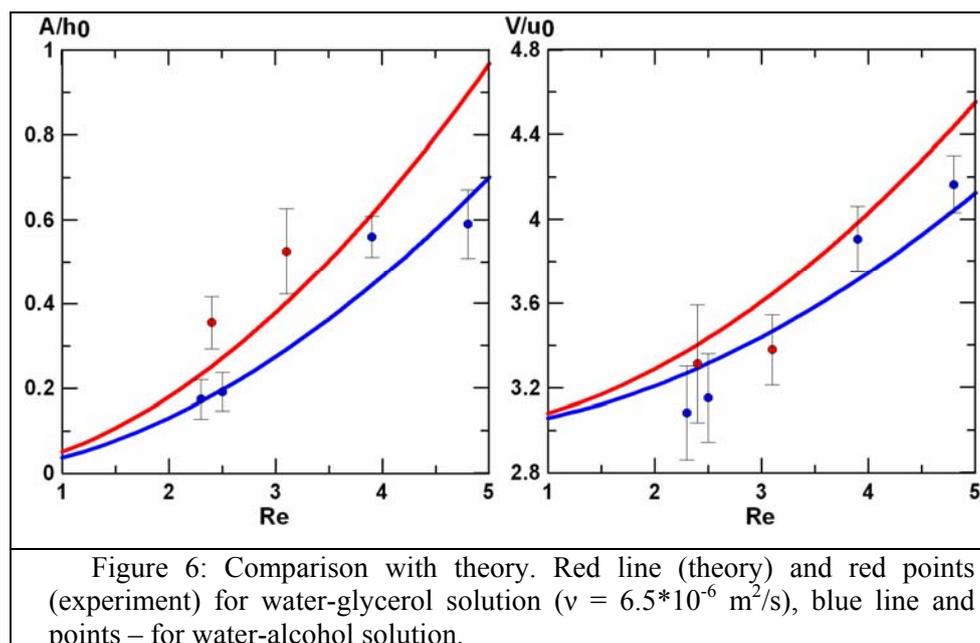
Spatial form and cross-section of stationary three-dimensional solitary waves for liquids with sufficiently different physical properties are shown in Fig.5. In general the shape of the waves is similar. But for more viscous liquid the cavity after the main peak and capillary precursor are smaller then for less viscous liquid.

Whereas the shape of registered stationary 3-D waves is similar to theoretically predicted on the basis of Kuramoto-Sivashinsky equation (KS) [5] when  $Re \sim 2$ , this shape sufficiently transforms and becomes similar to the shape, obtained [6] on the basis of the generalized Kuramoto-Sivashinsky equation (gKS) for  $Re > 3$ . Such stationary wave characteristics as amplitudes

and velocities are close to the theoretically predicted ones for experiments at low  $Re < 5$ . This can be seen in Fig.6, where dependency of amplitude and velocity on Reynolds number of stationary waves is shown in comparison with values predicted on the KS basis. Note that difference between KS and gKS values of amplitudes and velocities for this range of Reynolds numbers is very small and is essentially less than experimental error.

There exist another more common, but less pronounced distinctions between experimentally registered and theoretically predicted characteristics of stationary 3-D waves. For example, the ratio of the wave amplitude to the value of depression behind wave hump for theoretically predicted stationary waves is approximately equal to 9 for a wide range of  $Re$  [6] whereas for experimentally registered waves this ratio diminishes monotonically with growth of  $Re$  and appears to be smaller approximately by factor of two.





## Conclusions

Experimental results on evolution of 3-D solitary waves on falling liquid films for Reynolds numbers of film flow with  $Re < 25$  are presented. LIF method is applied to measure the shape and instant velocity of the waves. Generation of the wave train as well as evolution in the form of one-humped solitary wave is main scenarios of the wave evolution.

Stationary 3-D solitary waves were registered for several experimental conditions. For low Reynolds numbers their amplitude and velocity agree well with theoretically predicted values. But there exist some differences between shape of theoretically predicted and registered stationary waves, especially in the regions of capillary precursor and tail part.

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