# THE RIEMANN-TYPE SOLUTION FOR NONADIABATIC GAS IN THE GRAVITATIONAL FIELD

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Propagation of nondissipative nonlinear one-dimensional perturbations in homogeneous gas can be exactly described by the Riemann solution [1]. In [2] it is shown that Riemann-type solution also describes vertical propagation of such perturbations in presence of uniform gravitational field. It is essential that solutions presented in [2] need the assumption that not only propagation but also initial state of medium are adiabatic.

It is of interest to find exact solution of the one-dimensional hydrodynamic equations for perturbations occurring in the mechanically-equilibrium nonadiabatic initial state of stratified gas in gravitational field [3]. In this case, it is impossible to use the assumption of barotropy $p = p(\rho)$ [4] and, as the result, the analysis of system of hydrodynamic equations in the Euler approach becomes complicated. It is expedient to use the Lagrange approach at which (instead of the system of equations for pressure $p$, density $\rho$ and speed $v$) the closed equations of the second order can be deduced for each of variables: $z(a, t)$, $p(a, t)$ and $\rho(a, t)$, where $z$ and $a$ are the Euler and Lagrange coordinates respectively.

Let us assume that at the initial moment

$$z(a,0) = a, \quad p(a,0) = p_0(a), \quad \rho(a,0) = \rho_0(a). \tag{1}$$

In the Lagrange approach for adiabatic processes in perfect gas (ideal gas with $\gamma = \dfrac{C_p}{C_v} = const$; terminology of [4]) in the one-dimensional case we have

$$\rho(a,t) = \rho_0(a)\left(\frac{\partial z}{\partial a}\right)_t^{-1}, \frac{p(a,t)}{p_0(a)} = \left[\frac{\rho(a,t)}{\rho_0(a)}\right]^{\gamma}, \left(\frac{\partial^2 z}{\partial t^2}\right)_a = -\frac{1}{\rho(a,t)}\left(\frac{\partial p}{\partial a}\right)_t\left(\frac{\partial z}{\partial a}\right)_t^{-1} + g_z(z). \tag{2}$$

If the initial state is mechanically-equilibrium, the relation $\dfrac{dp_0(a)}{da} = \rho_0(a) g_z(z)$ takes place and thus, from (1), (2) it follows:

$$\left(\frac{\partial^2 z}{\partial t^2}\right)_a - \frac{c_0^2(a)}{(\partial z/\partial a)_t^{\gamma+1}}\left(\frac{\partial^2 z}{\partial a^2}\right)_t = g_z(z) - \frac{g_z(a)}{(\partial z/\partial a)_t^{\gamma}}, \tag{3}$$

where $c_0^2(a) = \dfrac{\gamma p_0(a)}{\rho_0(a)}$. In a homogeneous gravitational field, i.e. at $g_z = const$, equation (3) take the form (compare with [5])

$$\ddot{\xi} - \frac{c_0^2(a)}{(1+\xi')^{\gamma+1}}\xi'' = g_z\left[1 - (1+\xi')^{-\gamma}\right], \tag{4}$$

where $\dot{f} = \left(\dfrac{\partial f}{\partial t}\right)_a$, $f' = \left(\dfrac{\partial f}{\partial a}\right)_t$ and the variable $\xi(a, t) = z(a, t) - a$ describes displacement of particles of medium.

At the same time for the function $\theta(a, t)$, directly related to $\xi'$, $p$ and $\rho$:

$$\theta(a,t)=\left(\frac{\partial z}{\partial a}\right)_t=1+\xi'=\frac{\rho_0(a)}{\rho(a,t)}=\left[\frac{p_0(a)}{p(a,t)}\right]^{\frac{1}{\gamma}}, \tag{5}$$

after differentiation (3) with respect to $a$ (at $g_z = const$), we receive:

$$\ddot{\theta}-\frac{c_0^{\,2}(a)}{\theta^{\gamma+1}}\theta''=\left[\left(c_0^{\,2}\right)'+\gamma g_z\right]\frac{\theta'}{\theta^{\gamma+1}}-(\gamma+1)\frac{c_0^{\,2}(a)}{\theta^{\gamma+1}}\theta'\theta'. \tag{6}$$

Now we shall try to find solution at which the Lagrange velocity of propagation of a preset value of variables $\theta$, $\xi'$, $p/p_o$ and $\rho/\rho_o$ depends not only on this value (as in the Riemann wave) but also explicitly on the Lagrange coordinate *a.* Thus, we shall search for the first integral of (6) in the form of

$$\left(\frac{\partial\theta}{\partial t}\right)_a=\beta(\theta,a)\left(\frac{\partial\theta}{\partial a}\right)_t. \tag{7}$$

Calculating $\left(\frac{\partial^2\theta}{\partial t^2}\right)_a$ by (7) and comparing the result with (6), one can see that (7) will be the first integral of equation (6), if the initial state have the constant gradient of temperature $T_0'=-2g_z/R$, ($R=R_u/\mu$, $R_u$ is the universal gas constant, $\mu$ is the molar mass) directed upwards, so this state will be not only mechanically-equilibrium, but also convective-stable. Besides, we obtain that

$$\beta(\theta,a)=\pm c_0(a)\theta^{-\frac{\gamma+1}{2\gamma}}, \qquad \text{where} \qquad c_0(a)=c_0(0)\sqrt{1-\frac{2g_z a}{RT_0(0)}}\,.$$

As the result, for any of variables $\theta$, $\xi'$, $p/p_0$ and $\rho/\rho_0$, the exact solution (generalized Riemann wave) can be deduced. For example, the relation for relative pressure $\Pi(a,t)=p(a,t)/p_0(a)$ take form: $\Pi=F\left[t-b(a)/\eta(\Pi)\right]$, where $F$ is the arbitrary function, $b(a)=\frac{RT_0(0)}{g_z}\left(1-\sqrt{1-\frac{2g_z a}{RT_0(0)}}\right)$, $\eta(\Pi)=c_0(0)\Pi^{\frac{\gamma+1}{2\gamma}}$.

Particularly, this solution allows us to find the moment of time and the distance from source, at which a shock wave arises, taking into account an influence of gravitation and temperature gradient.

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