DIRECT NUMERICAL SIMULATION OF THE HYDRODYNAMICAL PERTURBATIONS EVOLUTION

M.F. IVANOV, A.D. KIVERIN[c]

Joint Institute for High Temperatures of RAS, Moscow, 125412, Russia

[c]Corresponding author: Tel.: +79265335989; Email: alexeykiverin@gmail.com

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ABSTRACT: The evolution of gasdynamical flow perturbations is analyzed using a direct numerical simulations of the dynamics of viscous compressible gas. The study is based on the example of the forced axisymmetric flows in a chamber under a moving piston. The correlation analysis is applied to the set of identical calculations whose initial conditions were perturbed with a random velocity field of small amplitude. Obtained patterns of the perturbations evolution in the forced flow are compared with those obtained for the perturbations evolution inside two- and three-dimensional closed volumes.

INTRODUCTION

The studies of perturbations evolution and turbulence origins in the gas flows remain relevant for a decades. Studying the features of the dynamical processes can provide an information on the turbulent flows classification and their roles in these processes. Thus a developed phase of turbulence corresponds to the homogeneous isotropic turbulence for which there are no structure formation of the perturbations by definition. At the same time the dynamics of axisymmetric turbulence [1, 2] on the stage previous to the isotropic does not contradict to the formation of sustained connections between perturbations. This paper is devoted to reveal and analyze the emergence and evolution of such connections. The study is based on the example of the axisymmetric flow in a chamber under a moving piston and carried out using a direct numerical simulations.

There are two main approaches for studying complex gasdynamical flows: spectral and correlation. The first one allows to track evolution of the spatial structures of the flow, the second one allows to identify the interconnections between perturbations and their synchronization which give a clear pattern of the structure formation in the flow field. In the present paper for analysis of the gasdynamical perturbations field we chose a correlation approach which has an additional advantage in this case allowing to obtain and compare results on the different spatial scales (the scale along the axis is changing while the piston is moving). To assess the validity of the carried analysis the numerical simulations represents a setup of the experiment by Breuer, Oberlack and Peters [3].

EXPERIMENTS ON AXISYMMETRIC TURBULENCE AND PROBLEM SETUP

In [3] a set of experimental data was statistically analyzed to study statistical characteristics of the axisymmetric turbulence. The set of experimental data represents a set of measurements of the flow field using particle image velocimetry (PIV) during the consecutive compression strokes in the cylindric chamber with engine speed of 2000rpm. Measurements were carried out in a 1.6L four cylinder engine filed with air. Instantaneous velocity fluctuations, correlation parameters and integral statistical values were calculated using the instantaneous flow field and the flow field averaged over the set of compression strokes. In axisymmetric coordinate system the equations for the integral statistical values are as follows:

$$K = \frac{1}{V}\int\left(v_r^2 + \frac{1}{2}v_z^2\right)dV \tag{1}$$

$$\Lambda_{\alpha\alpha} = \frac{3}{4\cdot K}\int R_{\alpha\alpha}(l,m)\frac{1}{l^2+m^2}\cdot dl\cdot dm \tag{2}$$

$$R_{\alpha\alpha}(l,m) = \frac{1}{|G'|}\int_{G'} v_\alpha(r,z)\cdot v_\alpha(r+l,z+m)\cdot dr\cdot dz \tag{3}$$

where α=r, z - is one of two coordinates, K - turbulent kinetic energy, Λ_{rr}, Λ_{zz} - integral length scales along the radius and the axis correspondingly. In eq. (2) the integration is taken over the observation area, in eq. (3) G' - is an area for which (r,z) and (r+l,z+m) points belong to the observation area. In [3] the integration over the volume was replaced by the sum over the tracer particles. The results of correlation analysis were presented as a dependencies of integral length scales from the crank angle (CA) or piston position.

For numerical simulations of the processes observed in the experiments the following problem setup was accepted. The chosen geometry is a cylinder of radius R=0.04m and height H=0.09m, typical for automobile engines and volumetrically equal to the cylinder used in experiments [3]. The cylinder is initially filled with air at normal temperature and under atmospheric pressure (T_0=300K, p_0=1atm). The piston moves out from the bottom dead center (the point corresponding to the CA=0^O) according to the typical law for automobile combustors [4]:

$$U_p = U_0 \sin(2\pi\nu t)\left(1 + \frac{\cos(2\pi\nu t)}{\sqrt{11{,}8906 - \sin^2(2\pi\nu t)}}\right) \quad (4)$$

where ν - is a rotation frequency of the crankshaft, U_0 - is a maximal piston speed for corresponding rotation frequency. Numerical simulations give the flow field in the whole domain between piston surface and opposite end of the cylinder, however the correlation analysis is implemented only towards the flow field in the area corresponding to the top dead center as it was made in experiment [3].

MATHEMATICAL MODEL AND NUMERICAL METHOD

The gasdynamics of the air in the chamber under the moving piston is modeled using a traditional mathematical model for transport of viscous thermal conductive compressible gas (see e.g. [5] or [6] where a combustion model is included). The equations of state are calculated using polynomial interpolations of the JANAF data [4]. The viscosity and thermal conduction coefficients are calculated according to the data from [7]. Boundary conditions are adiabatic non-slip walls. It should be mentioned that non-slip conditions are principal for reproducing the shear flows causing the turbulization of the flow under piston.

Numerical method is based on splitting of the Eulerian and Lagrangian stages and known as coarse particle method (CPM) [8]. Detailed description of the modified CPM for optimal approximation scheme and details of the equations and transport coefficients were published in [5, 6]. The method was thoroughly tested and successfully used in many practical applications (see [5, 6] and cites within).

The piston movement was modeled using the merging of the computational cells along the axis. The gas parameters in the merging cells were calculated according to the laws of adiabatic compression or expansion corresponding to the phase of piston movement.

Practically in [3] the correlation analysis was implemented for velocity fluctuations defined as deviation of instantaneous velocity values from the mean values. Where these mean values were obtained using the averaging of the data obtained in the set of compression stroke experiments. At the same time a numerical simulations of the gasdynamical flows is usually used for space-time analysis of the instantaneous flow characteristics and does not give an information about the evolution of flow perturbations which is necessary for further correlation analysis. In paper [5] which results are partially included in this paper the following approach was proposed. The velocity perturbations were calculated from the set of computations with slightly different initial conditions. The initial conditions for one computation represented a random velocity field. Both velocity components were uniformly distributed on the interval (-A, A), where A - is a maximum amplitude of the perturbation which was less than 1 percent of the maximal piston speed. Thus an energy of the perturbations was sufficiently small compare with the characteristic energies of the process. After about 10 time-steps the flow field was corrected due to the mass conservation laws in the computational cells and the piston movement began. The flow field for every further time step was stored in memory. Further the results for every time step for not less than 10 computations were averaged and the mean flow field was obtained. The deviations of the velocities were obtained for every computational cell for every time step as:

$$\vec{V}(r,z,t) = \vec{U}(r,z,t) - \vec{U}'''(r,z,t) \quad (5)$$

where $\vec{U}(r,z,t)$ - is an instantaneous velocity, $\vec{U}'''(r,z,t)$ - is a mean velocity obtained using an averaging over the ensemble of computations.

The computations were carried out with a spatial resolution of $2 \cdot 10^{-4}$ - $4 \cdot 10^{-4}$m. The change of the cell size gave ~1% variation in thermodynamic parameters. Correct estimation of the velocity variation is difficult because of impossibility to get an equal random flow field for the cells of different sizes (the maximal variation in the velocity was about 1%), however the integral correlation values gave a good coincidence. It should me mentioned that Reynolds number on the scales of the cell size is about unity. Thus the computations with such a resolution guarantee the dissipation of turbulent pulsations on the scales which are much smaller than a characteristic scales of the problem.

CORRELATION ANALYSIS OF THE FLOWS IN THE CHAMBER UNDER PISTON

The method proposed in [5] has been applied to reproduce numerically the correlation analysis of the flows under the moving piston [3]. Figure 1 shows comparison of the numerical and experimental results.

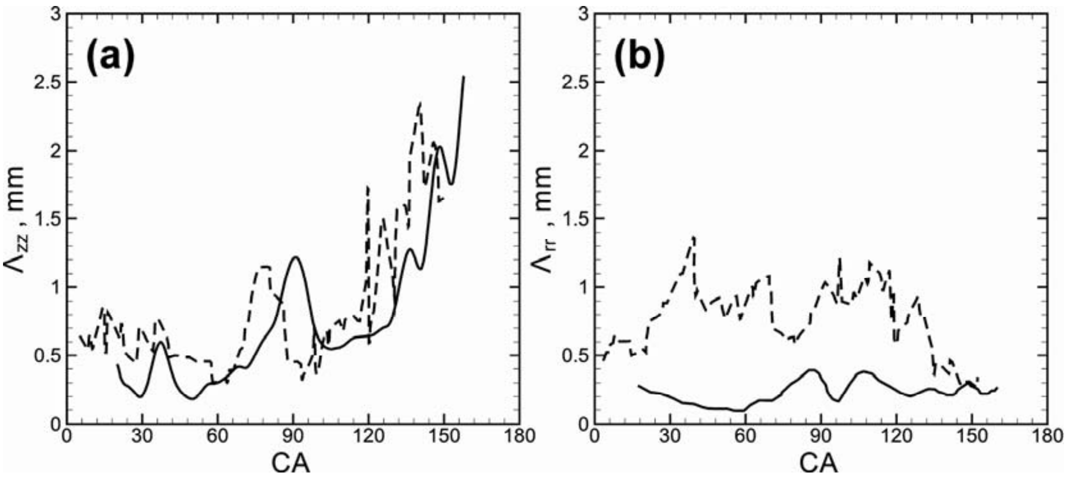

Fig. 1 - Evolution of integral length scales with crank angle during the compression stroke: axial Λ_{zz} (a) and radial Λ_{rr} (b). Dashed lines - experiment [3]. Solid lines - computations. Engine speed is 2000rpm.

From fig. 1 one can see that computational results obtained using the approach described above reproduce the experiment with sufficient detalization both qualitatively and quantitatively. There are some quantitative difference for radial integral scale however the qualitative pattern is similar (fig. 1b). In both cases (experimental and numerical) the curves can be treated as non-regular oscillations near corresponding mean values which are independent on a piston position. It should be mentioned that there are lag between experimental and numerical results which can be connected with a difference in a law for piston movement (unfortunately there are no such information for comparison in [3]). At the same time it was shown in [9] that root mean square velocity and kinetic energy of the perturbations are closely connected with a piston position. Therefore the difference in a piston speed may affect the integral length scale value.

The results presented above have a statistical nature. Therefore the choice of initial perturbation of the velocity field and the number of calculations in a set should affect the solution. The analysis of these factors was implemented in [5] and showed a sufficient stability of the obtained results towards them.

The results of [3, 5] show enhancement of the correlations while piston moves in the direction of the top dead center which in turn indicates the formation of stable structures. It is not intrinsic to the developed isotropic turbulence and defines features of axisymmetric flows which should be analyzed additionally. The information essential for detailed study of the emergence and evolution of the structures in the flow under the moving piston can be obtained from the visualization of the perturbations fields at the different time instants. Such analysis has not been done both in [3] and [5]. The flow fields of the velocity perturbations obtained numerically (eq. (5)) are shown in figure 2 for different positions of the piston. Crank angle values in fig. 2 from 2.4^O to 180^O correspond to the compression stroke and from 240^O to 359^O - to the expansion stroke.

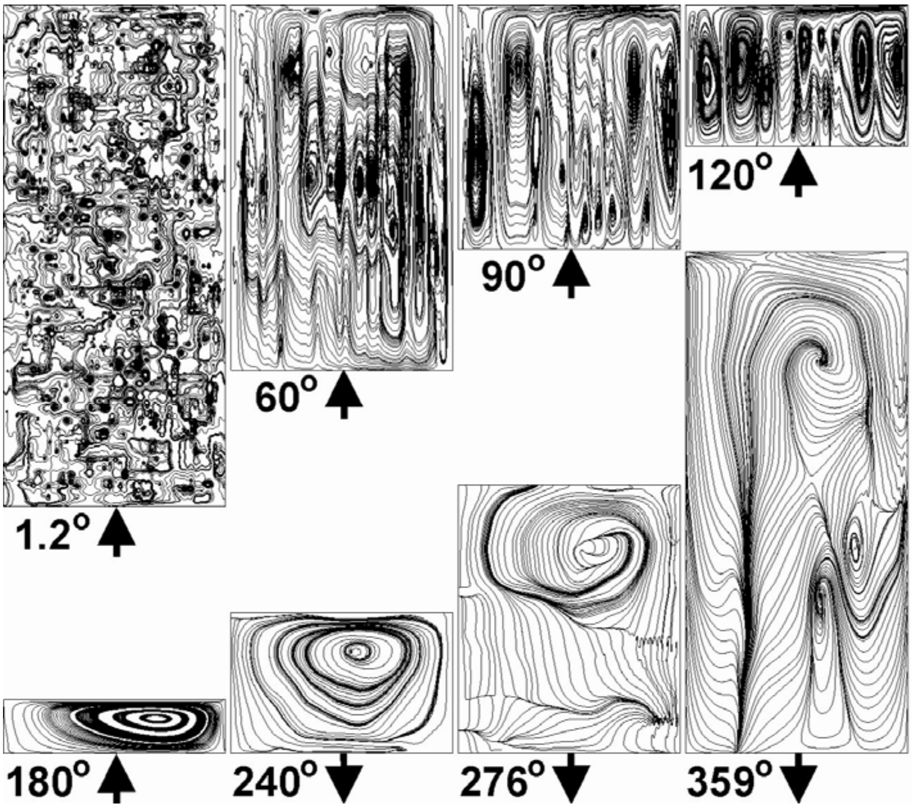

Fig. 2 Evolution of the velocity perturbations in cylindric chamber under the moving piston. Convoluted lines are the stream traces for the field of velocity perturbations. Numerals show corresponding crank angles. Arrows show the direction of the piston movement. Left boundary - symmetry axis. Right and upper bounds - are solid walls. Bottom boundary - is a piston surface. Engine speed is 2000rpm.

At the same time instants as shown in fig. 2 the stream traces for the flow fields represents identical slightly curved vertical lines going from the piston surface to the opposite end approximately during the whole phase of compression stroke. In the very vicinity to the upper piston position the flow field restructures into the large vortex occupying entire area between piston and opposite end. At the same time the perturbations of these flow fields creates a complex pattern permanently changing over time (figure 2). Out from the initial random field the perturbations evolve into the vortical structures tending to grow and merge with the neighbors (one can observe it already at the time instant corresponding to crank angle of 2.4^O). The process of vortices merging and formation of the larger vortices continues up to crank angle about $80\text{-}90^O$. Further process is defined by the evolution and interaction of the several large vortices which merge in a one large vortex on the final stage of the compression stroke. This large vortex totally defines the large-scale structure of the flow at this current time instant. Apparently the observed spike of the integral scale value Λ_{zz} (see fig. 1a) at crank angle of $80\text{-}90^O$ is connected with the reorganization of the interaction mechanism of the small-scale perturbations. The kinetic energy of the perturbations increases monotonically up to crank angle of 120^O and then decreases due to the deceleration of the flow on the late stage of compression stroke.

The expansion stroke was not studied earlier in [3] however numerical simulations [5] showed that on this phase the axial integral scale value Λ_{zz} increased and reached the level of saturation on the last stage of expansion phase. This level was about 25 times larger than that obtained at crank angle 180^O. In fig. 2 one can see that the vortices emerging on this phase have a spatial scales limited by the chamber dimensions. At the same time the structure of the perturbations flow field unrelated with the structure on the previous phase of compression. In fact the vortex emerging at the crank angle of 180^O totally erases the information about previous perturbations dynamics. And the further process is determined by the evolution of this vortex.

SMALL-SCALE PERTURBATIONS EVOLUTION WITHOUT EXTERNAL IMPACT

The results above concern a forced axisymmetric flow which essentially is a three-dimensional flow neglecting the tangential perturbations. Apparently such a good agreement of the obtained results with corresponding physical experiments justifies a chosen approach at least for determining the integral statistical values of the flows under the moving piston. Further extended study of the problem is carrying out using direct numerical simulations of the flows under the moving piston in three dimensions. However some features of the perturbations evolution caused by gas compression and expansion can be analyzed using already obtained results. In particular, the question of interest is how the growth of the spatial scales during the evolution of the perturbations field under the moving piston reflects the features of the entropy vortical perturbations [10] in absence of the external impact.

To examine the behavior of the weak perturbations two and three-dimensional gasdynamical simulations of the random uncorrelated perturbations evolution were carried out. Figures 3 and 4 illustrate an evolution of the vortical structures in closed volume with non-slip walls in two dimensions (fig. 3) and three dimensions (fig. 4). One can see that in both cases perturbations evolve forming structures with larger spatial scales up to the scales comparable with the dimensions of the volume. In three-dimensional case a cone vortex belted with a transverse toroidal flow forms. The direction of the vortex axis is determined by the rotor of the random field and therefore it is random.

According to the obtained results one can conclude that there are a basic pattern for the evolution of the weak perturbed flow field independent on the spatial dimensionality of the problem: randomly oriented weak perturbations merge into vortical structures which corresponds to the dynamics of entropy vortices [10]. Further the spatial scales of these structures increase. It allows us to conclude that the observed formation of the vortical structures in the chamber under the moving piston primary is caused by a non-linear dynamics of vortical fields but not a synchronization of the flows generated by the piston moving in the medium.

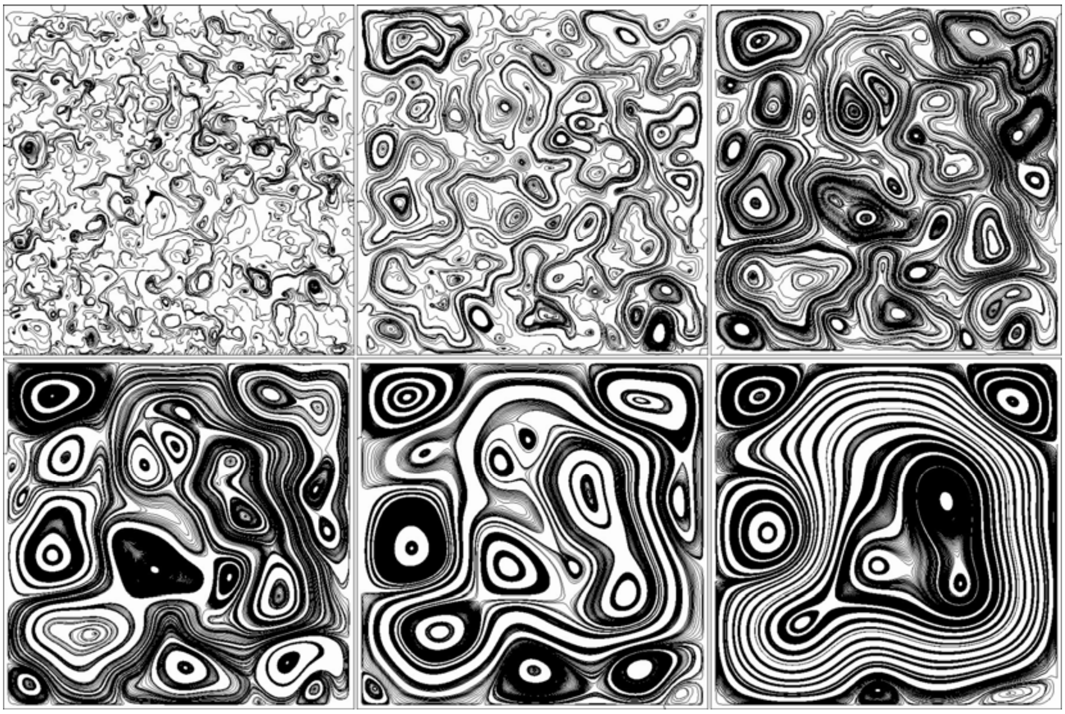

Fig. 3 Evolution of the velocity perturbations in closed volume with non-slip walls in two-dimensional planar geometry.

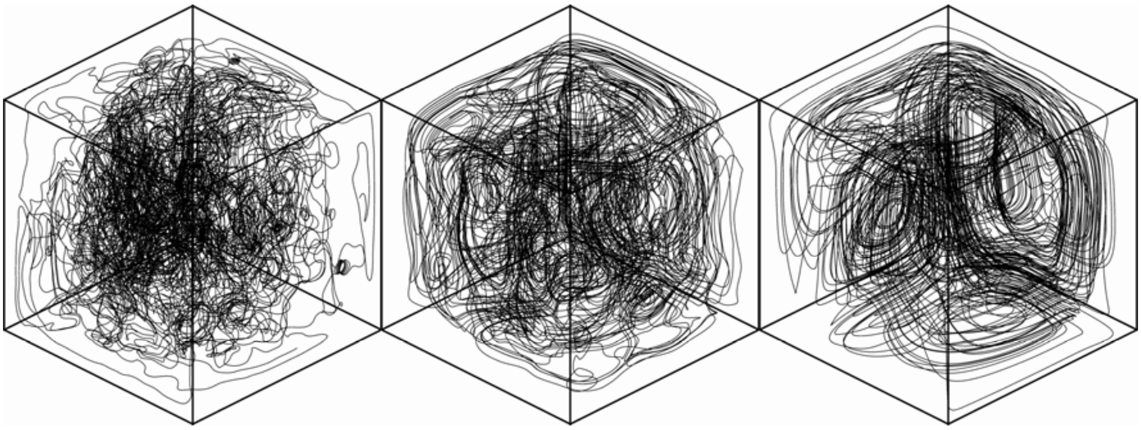

Fig. 4 Evolution of the velocity perturbations in closed volume with non-slip walls in three-dimensional geometry.

CONCLUSIONS

Developed approach of numerical modeling of the complex non-steady flows allows to extract and analyze small-scale perturbations of the flows. It has been applied to the problem of gas dynamics in the cylindric chamber under the moving piston providing us with a more detailed visualization of leading dynamical characteristics. The features of the perturbations evolution during the complete cycle of the engine are defined and described in details.

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