DIRECT AND INVERSE PROBLEMS OF HEAT TRANSFER MODELING IN THE SOIL

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ABSTRACT: The paper presents the solution of the problem of heat transfer in soil finite and semi-infinite thickness of the boundary conditions at the soil surface of the first kind with infinite harmonic. The developed methods determining the thermal diffusivity of the soil, based on the solution (with two harmonics at the soil surface) of the inverse problems of heat transfer equations. If the temperature of the soil surface during the day (year) can be expressed by a single harmonic, then we can find the value of reducing the amplitude of the diurnal temperature with depth or temperature wave phase lag at different depths. This definition allows for appreciable error due to the fact that the soil temperature is not always varies sinusoidal due to the variability meteorological conditions. The introduction of the second harmonic of a sinusoidal temperature dynamics equation approximates the temperature variation of the active surface of the soil to the real picture. Solution of the heat conduction with infiltration problem is suggested. This solution permits to determine the soil term diffusivity using some data of the quotidian variability of soil temperature under infiltration conditions.

INTRODUCTION. The current management of the water and thermal regimes of soils is based on the polyvariant prediction calculations of water and thermal regimes and the choice of the optimum impact on the soil cover for their improvement. These calculations are based on the use of mathematical models of the heat and water transfer in the soil. An obligatory experimental support for the physically sound models of the thermal regime is the function of the soil thermal diffusivity, i.e., the thermal diffusivity as a function of the soil water content, which depends on the composition and properties of the soil horizons. The problem is to develop simple, exact, rapid, and scientifically sound methods for acquiring this essential thermo physical parameter. The aim of this work was to find the mean integral solution of the model for heat transfer in soil based on the solution of the inverse problems and to substantiate the experimental methods for the determination of the soil thermal diffusivity coefficient on the basis of this solution. The purpose of this study is to develop mean integral solutions of the heat transfer in soil, based on solving inverse problems, and in the rationale of the experimental methods on the basis of this decision, for determining the thermal conductivity of the soil.

THEORETICAL APPROACHES. To calculate the distribution of temperature field in the soil, as a rule, use the classical equation of heat transfer (Carlslaw, Jaeger, 1964; Nerpin, Chudnovsky, 1967; et al.):

$$c_v\left(x,t\right)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left[\lambda\left(x,t\right)\frac{\partial \mathrm{T}}{\partial x}\right] \qquad (1)$$

where $T\left(x,t\right)-$ the soil temperature at the point x of time t, $\lambda-$ the thermal conductivity, $c_v=\rho_b\,c_m-$ volumetric heat capacity, ρ_b- density of the soil, c_m- specific heat.

It is known that in simulation of soil processes in the soil-plant-atmosphere system is to identify the main stages (choice of equations describing these processes and boundary conditions) and realization (i.e. direct and inverse methods) models. Therefore, the systems approach analysis of the temperature regime of the soil is sometimes forced to resort to a simplification in the equation of heat transfer, and in the formulation of boundary conditions (Carlslaw, Jaeger, 1964; Mikayilov, 2007; et al.). For a number of tasks can make significant simplification of equation (1), if we take a constant coefficient of heat capacity and the coefficients of heat and temperature - are linearly varying with depth of soil or permanent. However, such simplification for the majority of soils is virtually unacceptable.

Formulation and solution of various problems of heat transfer in soils are described in detail (Carlslaw, Jaeger, 1964, Mikayilov, 2010; et al.). For example, to address both the direct problem of heat transfer in soil (the prediction of heat transfer in the soil) and inverse (determination of thermal diffusivity according to the field or laboratory experiments), equation (1) mainly to simplify the form:

$$\frac{\partial T}{\partial t} = \kappa \cdot \frac{\partial^2 T}{\partial x^2} \text{ , where } \left(\kappa = \lambda / c_v\right) \tag{2}$$

and consider its decision, obtained under different boundary and the initial conditions. The initial conditions reflect the state of the state variable to zero (initial) time. When formulating the problem to a theoretical description of the problem of the quasi-stationary regime (e.g., daily or annual variations in soil temperature) initial condition is absent (so-called "problem without initial conditions"). The possibility of such a formulation of the problem follows from the experimental determination of the existence of strict periodicity in the daily and annual rhythms of temperature and can be assumed that any given instant is removed from the "start" the process by an infinitely long period of time. Naturally, the notion of "beginning" and "infinite duration" of the process rather conventional and express the generalization of experience, claiming that under natural conditions, soil temperature is expressed by a periodic function of time in the form of a harmonic curve with a finite number of harmonics that are multiples of the basic (annual or daily) (Mikayilov, 2010; Nerpin, Chudnovsky, 1967; et al.). Under the boundary conditions to understand the quantitative description of physical processes, the dynamics of properties on the top (as a rule, on the soil surface) and bottom (at some specific depth) boundaries of the soil stratum. There are conditions for the 1st and 2nd kind. The boundary condition on the surface of the 1st kind recorded in the usual form

$$T(0,t) = \varphi(t) \tag{3}$$

where the function $\varphi(t)-$ is the surface temperature of the soil from time to time, an analytical expression which must be defined in advance. It should be noted that for the analysis and comparative assessment of various factors on the temperature distribution in the soil, the heat flow and heat accumulation in the soil is best to deal with the simple structure. The simplest form of these formulas is, when given the time evolution of temperature on the surface. Therefore, the boundary conditions of 1st kind are usually preferred. If it is determined that the change in soil temperature at the surface is periodic, then the analytical expression functions $\varphi(t)$ take:

$$\varphi(t) = T_0 + T_a \cdot \cos(\omega t + \varepsilon) \text{ Or } \varphi(t) = T_0 + T_a \cdot \sin(\omega t + \varepsilon) \tag{4}$$

If the temperature of the soil surface is a periodic function with period $\tau_0 = 2\pi/\omega$, $\varphi(t)$ is decomposed in a Fourier series, and the 1st boundary condition on the surface in this case will look like:

$$T(0,t) = T_0 + \sum_{j=1}^{m} T_j \cdot \cos\left(j\omega t + \varepsilon_j\right) \tag{5}$$

Here, T_0- the average daily (or annual) temperature of the active surface of the soil, T_a- the amplitude of temperature fluctuations of active soil surface, $\omega = 2\pi/\tau_0-$ circular daily (or annual) frequency, τ_0- the period of the temperature wave, expressed in days or years, $\varepsilon-$ the phase shift, depending on the time origin, $m-$ the number of harmonics.

1st boundary conditions should be used when we are interested in heat transfer processes within the soil stratum, defined stratification of the soil profile, structural heterogeneity, and the thermal situation near the soil surface is considered as a background against which these processes are studied. This situation applies, for example, analysis of thermal effects associated with loosening or compaction of the surface layers of soil, surface peat with sand, use of mulch of loose materials and many other phenomena and processes. In those cases where we are interested in the influence of components of the heat balance of the active surface of the soil on the thermal regime of the soil, usually applied boundary condition of the second kind ("Theories and Methods…", 2007, et al.):

$$-\lambda \frac{\partial T(0,t)}{\partial x} = \psi(t) \tag{6}$$

where $\psi(t)-$ the algebraic sum of radiation, turbulent heat fluxes and evapotranspiration; in the calculation of these quantities are specified. However, condition 2 of the second kind (condition 2) does not identify the role and influence on the temperature field in the soil of individual meteorological parameters, as in (6) in explicit form, they are not included.

Some advantages of the boundary conditions for the use of the 3rd kind (Nerpin, Chudnovsky, 1975):

$$x = 0: \ -\lambda \frac{\partial T(0,t)}{\partial x} = \alpha\left[T_{\text{sur}}(t) - T(0,t)\right] \tag{7}$$

where $\alpha -$ the coefficient of convective heat transfer, $T_{sur}(t) -$ air temperature, $T(0,t) -$ surface temperature of the soil.

It may be noted that the use of condition 3 in this form can be used only when the convective heat transfer substantially predominates over other types of heat transfer at the boundary of the soil - the environment, such as high wind speed. In addition to these linear boundary value problems, also raises problems with nonlinear boundary (4th order) conditions:

when $$\lambda \frac{\partial T(0,t)}{\partial x} = \sigma\left[T^4(0,t) - T^4{}_{sur}(t)\right] \quad (8)$$

In soil physics, it is necessary to specify the considered thickness and condition at the lower boundary. This necessity is dictated by the fact that the calculations necessary to reduce the balance of heat or substances. In this regard, it is important to determine the estimated thickness of the soil and the condition at the lower boundary. Usually the soil is treated as an array of half-closed, and the lower boundary condition is given by taking into account the fact that the soil temperature at great depth is constant and equal in this case put the boundary condition 1 of the second kind:

$$T(\infty,t) = T_0 \quad (9)$$

or temperature fluctuations decay rapidly with depth and $x > L$ at a certain depth, soil temperature at a predetermined interval of time is practically unchanged. This allows instead of (9) used as the boundary condition 1 (the 1st-order condition or the condition of the first kind):

$$T(L,t) = T_0 \quad (10)$$

The lower boundary condition 1 is applicable for automorphic soils of arid and semiarid areas, where on the depth 3-5 m the temperature in real time (i.e. one year or several years) can be considered practically constant. In some cases, when modeled by a relatively short temporal period, this condition is applicable for the hydromorphic soils, and in general for soil humid landscapes. However, the depth of this border should be a few meters and for its selection should be taken into account peculiarities of the soil, terrain, possible heat flow in the lower part of the soil profile.
If there is no heat flux at depth should stipulate:

$$\frac{\partial T(L,t)}{\partial x} = 0 \quad (11)$$

Boundary condition 4 is usually applied when the soil is treated as a multilayer system. They are formulated in the form of equality of temperatures and heat flux continuity at the boundary layers:

$$T(x,t)\big|_{x=l_i-0} = T(x,t)\big|_{x=l_i+0}, \quad \lambda_i \frac{\partial T(x,t)}{\partial x}\bigg|_{x=l_i-0} = \lambda_{i+1}\frac{\partial T(x,t)}{\partial x}\bigg|_{x=l_i+0} \quad (12)$$

where $l_i -$ the boundary layers $(i = 1,2,3,...n)$, $l_i + 0$ and $l_i - 0$ consequently the upper and lower boundaries of the soil layers l_i. Such conditions should be used in layered soils, in soils on binary, etc. Knowing the initial and boundary conditions, we can offer various solutions of the direct problem of heat transfer, that is, finding the dynamics of the temperature at a given depth. In particular, it is of great practical interest is the problem of thermal conductivity in the soil with a periodically varying temperature at the surface.

THE SOLUTION OF THE DIRECT PROBLEM OF HEAT TRANSFER IN THE SOIL. Solution of equation (2) obtained without and with initial conditions (5) and (9) in dimensionless variables has the form:

$$\mathrm{T}(y,\tau) = \mathrm{T}_0 + \sum_{j=1}^{m} \Phi_j(y,b_j)\cdot\cos\left[j\bar{\omega}\tau + \alpha_j(y,b_j)\right] \quad (13)$$

where $y = x/L$, $\tau = \kappa t/L^2$, $b_j = \sqrt{j\bar{\omega}/2}$, $\bar{\omega} = \omega L^2/\kappa$ and

$$\Phi_j(y,b_j) = \mathrm{T}_j \cdot e^{-b_j y}, \quad \alpha_j(y,b_j) = \varepsilon_j - \psi_j(y,b_j), \quad \psi_j(y,b) = b_j\, y \quad (14)$$

This problem has been studied Fourier, and in the studies of Kelvin (Carslaw, Jaeger, 1964, pp. 85-87) was first applied in determining the temperature fluctuations of soil Edinburgh.

To our opinion, the decision is possible without initial conditions for $T(L,t) = T_0$ and $\partial T(L,t)/\partial x = 0$. For the calculation and prediction of soil temperature $T(x,t)$, as well as the definition of the parameter model (2), commonly used solution of the heat equation, obtained under the condition that the soil temperature at infinity is constant, that is the case: $T(\infty,t) = T_0$. However, in carrying out practical calculations is not possible as the original data set values of soil temperature at infinity, because they are unknown. Therefore, usually in such cases, instead of *T (∞, t)* is given by the temperature at a certain depth *L*, from which, when *x*>*L* the value of *T* (*x*, *t*) =const or ∂*T* (*L*, *t*)/∂*x*=0. Thus, the actual conditions more in line not a boundary condition (9) and condition (10) or (11). Therefore, you should consider the boundary problem (2), (5) and (10) or (2), (5) and (11). We can show that their decision also has the form (13), where the functions $\Phi_j\left(y,b_j\right)$ and $\psi_j\left(y,b_j\right)$ under the boundary conditions (5) and (10), to be determined by:

$$\Phi_j\left(b_j,y\right)=\mathrm{T}_j\cdot\mathrm{K}_j^-\left(b_j,y\right),\ \psi_j\left(y,b_j\right)=\mathbf{arctan}\left[\mathrm{P}_{2j}^-\left(y,b_j\right)/\mathrm{P}_{1j}^-\left(y,b_j\right)\right] \quad (15)$$

and the boundary conditions (5) and (11) are defined through:

$$\Phi_j\left(b_j,y\right)=\mathrm{T}_j\cdot\mathrm{K}_j^+\left(b_j,y\right),\ \psi_j\left(y,b_j\right)=\mathbf{arctan}\left[\mathrm{P}_{2j}^+\left(y,b_j\right)/\mathrm{P}_{1j}^+\left(y,b_j\right)\right] \quad (16)$$

where,
$$\mathrm{K}_j^\pm\left(b_j,y\right)=\sqrt{\frac{\mathbf{ch}\left(d_j\right)\pm\cos\left(d_j\right)}{\mathbf{ch}\left(2b_j\right)\pm\cos\left(2b_j\right)}},d_j=2b_j(1-y)\ \ q_j=b_j\left(2-y\right) \quad (17)$$

$$\mathrm{P}_{1j}^\pm=\mathbf{ch}\left(q_j\right)\cos\left(b_jy\right)\pm\mathbf{ch}\left(b_jy\right)\cos\left(q_j\right),\ \mathrm{P}_{2j}^\pm=\mathbf{sh}\left(q_j\right)\sin\left(b_jy\right)\pm\mathbf{sh}\left(b_jy\right)\sin\left(q_j\right) \quad (18)$$

$\mathrm{ch}\left(z\right)=\left(e^z+e^{-z}\right)/2$, $\mathrm{sh}\left(z\right)=\left(e^z-e^{-z}\right)/2-$ the hyperbolic cosine and sine respectively.

Based on these decisions (for $m=1$), i.e. (14) - (16), drawn up following a nomogram (Fig. 1), by which the coefficient of thermo diffusivity (κ) is determined for an arbitrarily chosen dimensionless depth $y=x/L$.

THE SOLUTION OF THE INVERSE PROBLEM OF HEAT TRANSFER IN THE SOIL. If the temperature of the soil surface during the day (year) can be expressed by one harmonic, then the thermal diffusivity *k* can be found from the magnitude of reduction of the daily amplitude of temperature with depth or on the base of lag phase of the temperature wave at different depths [1-3, 5-6].

This definition allows for appreciable errors due to the fact that the soil temperature is not always varies by the sinusoidal way. The introduction of the second harmonic (5) approximates the temperature variation of the active surface of the soil to the real picture. Using the solution of (4.1) for $m=2$, one can derive a formula for determining the thermal diffusivity κ for an arbitrary period τ_0 and the dimensionless depth y. To do this one needs t_0 know the temperature distribution in the soil layer $\left[0,L\right]$ for eight points of time on the calculated interval of time τ_0.

Next, using this solution (13) for $m=2$, first for an arbitrary depth y and time $t_i=i\cdot\tau_0/8$ we should write the following equation:

$$\mathrm{T}\left(y,t_i\right)=\mathrm{T}_0+\Phi_1\cdot\cos\left(\frac{\pi}{4}i+\alpha_1\right)+\Phi_2\cdot\cos\left(\frac{\pi}{2}i+\alpha_2\right),\left(i=\overline{1,8}\right) \quad (19)$$

After some transformations of these equations we have:

$$\sum_{i=1}^{4}\left[\mathrm{T}\left(y,t_i\right)-\mathrm{T}\left(y,t_{i+4}\right)\right]^2=8\Phi_1^2\left(y,b_1\right) \quad (20)$$

Function on the right side of (20), i.e. $\Phi_1(y,b_1)$, depending on the boundary conditions, determined from (14) - (16) respectively. $\mathrm{T}(y,t_i)-$ the value of the temperature of soil in the dimensionless depth *y*, at time (i = 1, 2, 8). For example, if $\tau_0=24h$, then $t=3,6,9,..$ and an hour.

Using graphics for the function $\Phi_1(y,b_1)$, given in (Fig 1.), we can find the value of the parameter $b=b_*$, where the coefficient of thermal conductivity is determined by the formula:

$$\kappa^*=\frac{\pi\cdot L^2}{\tau_0\cdot b_*^2} \quad (5.4)$$

In contrast to previously developed methods [3], here for determining of the *k*, we need to know in advance the distribution of temperature $\mathrm{T}\left(y_*,t_i\right)$ over time in soil layer $\left[0,L\right]$ on an arbitrary dimensionless depth $y_*=x/L$ for the eight points of time, which allows determine the parameter *k* with higher accuracy.

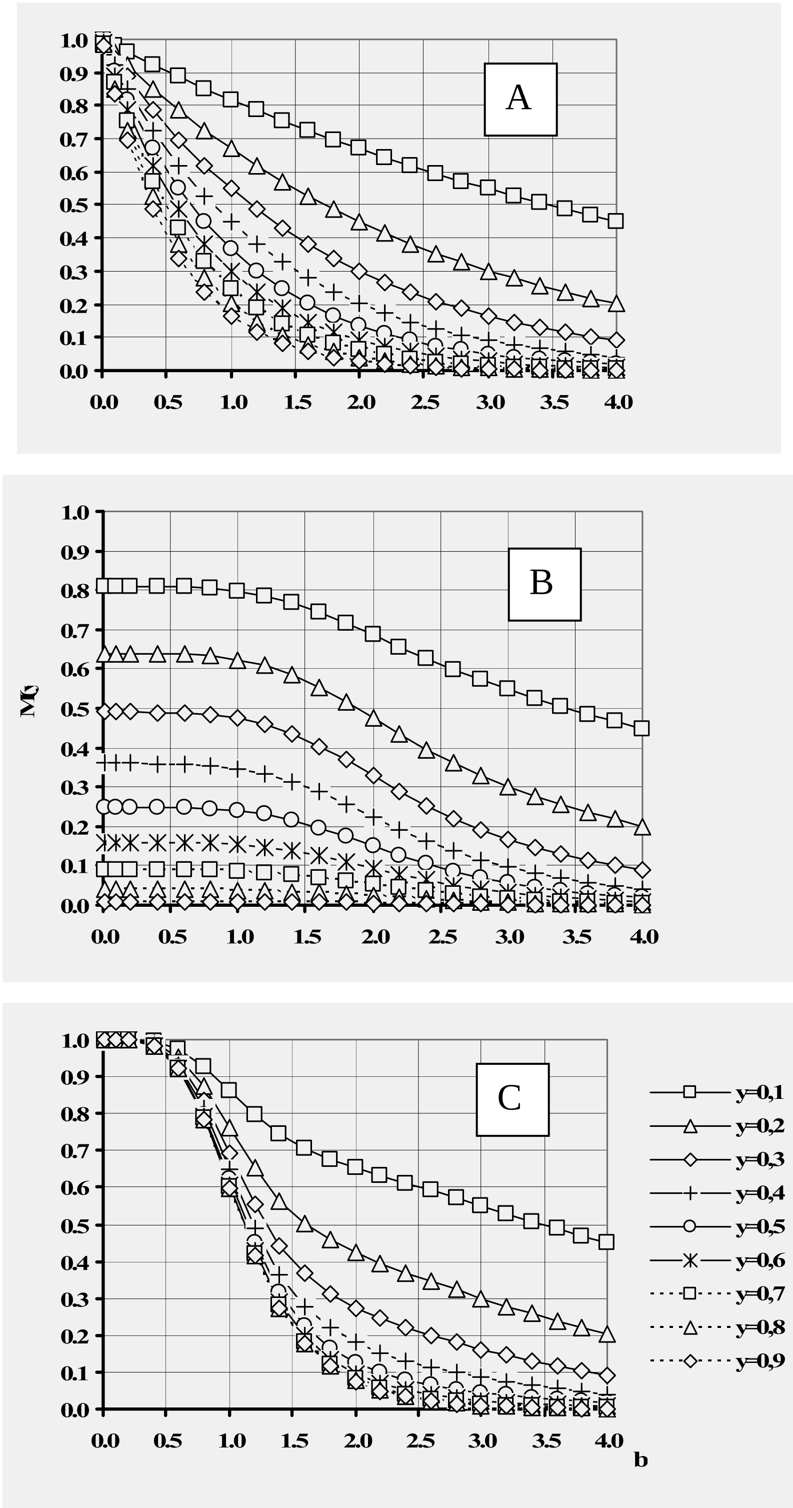

Fig 1. Graphs of the function **M(y, b)** at the lower boundary conditions of the first *(*A, B*)* and second (C) type at the lower boundary for **T (∞, t) =T$_0$** (A), with T **(L, t) = T$_0$** (B) and **∂T (L, t)/∂x=0** (C) to determine the value ***b*** and subsequent calculation of thermal diffusivity.

CONCLUSIONS. The proposed approaches and equations for determining the thermal diffusivity of soil layers are the theoretical basis for the development of experimental procedures for determining the thermal diffusivity of soil. The main problem, in our opinion, is to select the humidity range in which these methods and to determine the value. This requires significant periods of time, as the soil moisture changes in natural conditions is very slow. Testing these methods on various experimental data obtained in field and laboratory conditions, shows the possibility of using information about the diffusivity in the predictive mathematical models. This will increase the reliability, accuracy, adequacy and expand the boundaries of the use of predictive mathematical models to optimize the management of the thermal regime of natural and artificial soils in open and closed ground, increase the prediction accuracy of various thermal phenomena in the short-and long-term projections of climate change or soil conditions.

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